

ROCKY FLATS ENVIRONMENTAL TECHNOLOGY SITE

TBD-00163

**Technical Basis Document for Radiological
Characterization of Surface Contaminated Objects**

Revision 0

March 19, 2001

Reviewed For Classification/UNCI

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Date: March 19, 2001 UNU

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EXECUTIVE SUMMARY

This technical basis document (TBD) provides the basis for Radiological Safety Practice (RSP)-09.05, "Radiological Characterization for Surface Contaminated Objects." A Surface Contaminated Object (SCO) is a solid object which is itself not radioactive, but which has fixed and/or removable radioactive contamination distributed on any of its surfaces.

The characterization methods described here are used to determine:

- compliance with DOT regulations for categorization of SCO
- total radioactivity for all radionuclides within a shipping container packed with SCOs
- radioactivity concentration (activity per gram of waste) in the shipping container to demonstrate conformance to portions of disposal site waste acceptance criteria (WAC) for near-surface land disposal.

Two statistical methods used to characterize populations of objects that are candidates for SCO classification are defined. The first method, Plan A, has been implemented in procedures since 1998. However, Plan A employs non-parametric analysis and produces results that are sufficiently accurate to ensure compliance with shipping regulations and waste acceptance criteria. Because of built-in conservatism, Plan A includes a high potential for inappropriately failing surface contaminated objects that, in fact, may conform to shipping and waste acceptance criteria.

Plan B is newly defined in this document. Plan B is a parametric approach that includes tests for detecting significant non-normality of the sample data and for sufficiency of the sample size. After passing at least one of the tests, an upper tolerance limit (UTL) and the estimated mean are calculated. The UTL is used for comparison to the SCO limit. This method provides 90% confidence that at least 98% of the surfaces in the sampled population will be less than the UTL. The estimated mean is used to calculate package contents and specific activity. The result is a less conservative analysis that will enable more low-level radioactive waste to be appropriately shipped under the SCO classification for near-surface land disposal.

Methods for estimating surface area of waste container contents based on weight are defined, as are the methods for calculating total activity and activity concentration in a waste package.

Uncertainties of the characterization methods are considered. Plan A includes an intentional bias intended to overestimate both contamination level (a DOT compliance value) and specific activity (a disposal site waste acceptance criterion) to ensure neither are violated. This results in a conservative estimate of key quantities, and a wide uncertainty band around each estimate. For Plan A, combined relative uncertainty may be as little as 40%, more typically on the order of 120%, with the worst case being about 400%. For Plan B, which is not intentionally biased, the estimates are: best case

is 30%, typical is 40%, and worst case is 90%. Plan C uncertainties are similar to Plan B for best and typical case, and 60% for worst case.

1 INTRODUCTION

This technical basis document (TBD) defines the concepts implemented in Radiological Safety Practice (RSP)-09.05, "Radiological Characterization for Surface Contaminated Objects."¹ The TBD supercedes, "Radiological Field Characterization of Low Level Waste by Measuring Surface Contamination and Calculating Total and Specific Activity," RF/RMRS-98-242², which was effective June 22, 1998. The methods described here are used to determine

- compliance with DOT regulations for categorization of Surface Contaminated Objects (SCO)
- total radioactivity for all radionuclides within a shipping container packed with SCOs
- radioactivity concentration (activity per gram of waste) in the shipping container to demonstrate conformance to portions of disposal site waste acceptance criteria (WAC) for near-surface land disposal.

An SCO is a solid object which is itself not radioactive, but which has fixed and/or removable radioactive contamination distributed on any of its surfaces. Examples of SCOs that are being disposed as low-level radioactive waste during Rocky Flats Environmental Technology Site (Site) closure activities include: tools, desks, cabinets, computers, laboratory cabinets, bench tops, fume hoods, ducting, safes, bakeout ovens, vacuum cleaners and air movers with filter media removed, sinks, sheet metal, metal bar stock, piping, rigid plastics, wallboard, flooring, plastic sheeting, cardboard, light fixtures, and glovebox components. Books and paper, especially unbound paper, and cloth items may fit the definition of SCO, but are more likely to be well-characterized as Low Specific Activity (LSA) material.

If radioactive material is incorporated into the volume of an object through irradiation or absorption, or if a quantitative survey could not be made on an accessible surface of an item, then the waste does not qualify as SCO using the methods defined here. Instead it may be categorized as "Dry, Active Waste," LSA-II, in accordance with NUREG 1608 4.1.1. Objects with painted surfaces may be categorized if process knowledge and/or radiochemical analyses are used to evaluate the inaccessible (painted) area contamination levels.

It is not necessary to survey every object or potentially contaminated surface in order to characterize it. Knowledge of the process in which the objects were used, contamination levels where the objects are stored, and statistical sampling can all be used to infer the contamination levels for a population of items.

The statistical method RSP-09.05, Revisions 0 and 1 was based on a non-parametric statistical method, described in two memoranda,^{3,4}. The method, called Plan A in this report, includes calculation of total activity and activity concentration in the shipping container based on an extreme value estimator of the surface contamination level. The

non-parametric method, which is summarized in this report, is not altered, and it continues to be implemented in RSP-09.05.

An alternate parametric statistical method, called Plan B, is newly defined in this report. The alternate method is especially intended for use when contamination levels approach the U.S. Department of Transportation (DOT) Surface Contaminated Objects (SCO) limits, but could be used in any activity scenario. When this alternate statistical method is selected, the calculations of total activity and activity concentration in the shipping container use a mean estimator of surface contamination level. The new method is less conservative, yet continues to provide high confidence that the material meets the criteria for shipment under the appropriate SCO classification.

Other minor changes or additions include:

- an expanded table of surface area to mass ratios
- a calculation method for non-fixed contamination wiping efficiency
- a definition for a non-statistical method, called Plan C, for characterization when the entire surface of the object(s) is measured.

2. Regulations, Guidance, and Criteria

The regulations, guidance and criteria pertinent to this process are identified and summarized in this section.

2.1 SCO Definition from 49 CFR 173.403

“Surface Contaminated Object (SCO) means a solid object which is not itself radioactive but which has Class 7 (radioactive) material distributed on any of its surfaces. SCO must be in one of two groups with surface activity not exceeding the following limits:

- (1) SCO-I: A solid object on which:
 - (i) The non-fixed contamination on the accessible surface averaged over 300 cm² (or the area of the surface if less than 300 cm²) does not exceed 4 Bq/cm² (10⁻⁴ microcurie/cm²) for beta and gamma and low toxicity alpha emitters, or 0.4 Bq/cm² (10⁻⁵ microcurie/ cm²) for alpha emitters;
 - (ii) The fixed contamination on the accessible surface averaged over 300 cm² (or the area of the surface if less than 300 cm²) does not exceed 4 x 10⁴ Bq/cm² (1.0 microcurie/cm²) for beta and gamma and low toxicity alpha emitters, or 4 x 10³ Bq/cm² (0.1 microcurie/cm²) for all other alpha emitters; and
 - (iii) The non-fixed contamination plus the fixed contamination on the inaccessible surface averaged over 300 cm² (or the area of the surface if less than 300 cm²) does not exceed 4 x 10⁴ Bq/cm² (1 microcurie/cm²) for beta and gamma and low toxicity alpha emitters, or 4 x 10³ Bq/ cm² (0.1 microcurie/cm²) for all other alpha emitters.
- (2) SCO-II: A solid object on which the limits for SCO-I are exceeded and on which
 - (i) The non-fixed contamination on the accessible surface averaged over 300 cm² (or the area of the surface if less than 300 cm²) does not exceed 400 Bq/cm² (10⁻² microcurie/cm²) for beta and gamma and low toxicity alpha emitters or 40 Bq/cm² (10⁻³ microcurie/ cm²) for all other alpha emitters;
 - (ii) The fixed contamination on the accessible surface averaged over 300 cm² (or the area of the surface if less than 300 cm²) does not exceed 8 x 10⁵ Bq/cm² (20 microcurie/cm²) for beta and gamma and low toxicity alpha emitters, or 8 x 10⁴ Bq/cm² (2 microcuries/ cm²) for all other alpha emitters; and
 - (iii) The non-fixed contamination plus the fixed contamination on the inaccessible surface averaged over 300 cm² (or the area of the surface if less than 300 cm²) does not exceed 8 x 10⁵ Bq/cm² (20 microcuries/cm²) for beta and gamma and low toxicity alpha emitters, or 8 x 10⁴ Bq/cm² (2 microcuries/cm²) for all other alpha emitters.”

This definition is interpreted in Table 2-1 “SCO Upper Limits” for the materials likely to be encountered at RFETS:

Table 2-1. SCO Upper Limits for Radionuclides Typically Found at RFETS ^a.

Type of Contamination ^b	SCO I Limit (dpm/100 cm ²)	SCO II Limit (dpm/100 cm ²)
Plutonium or Enriched Uranium		
Plutonium or Enriched Uranium, non-fixed ^c on accessible surfaces	2400	240,000
Plutonium or Enriched Uranium, fixed on accessible surfaces	2.4×10^7	4.8×10^8
Plutonium or Enriched Uranium, non-fixed + fixed on inaccessible surfaces	2.4×10^7	4.8×10^8
Natural or Depleted Uranium		
Natural or Depleted Uranium, non-fixed on accessible surfaces	24,000	2,400,000
Natural or Depleted Uranium, fixed on accessible surfaces	2.4×10^8	4.8×10^9
Natural or Depleted Uranium, fixed + non-fixed on inaccessible surfaces	2.4×10^8	4.8×10^9

a. This table is derived from the definition of SCO in 49 CFR Part 173.403, using the conversion convention shown in 49 CFR Part 173.443, Table 11. The conversion convention is 0.4 Bq equals 22 dpm.

b. When the radionuclide(s) emit both alpha and beta/gamma radiation the limits for alpha and beta/gamma are applied separately. For example, if a 100 cm² depleted uranium wipe indicated 15,000 dpm alpha and 15,000 dpm beta/gamma, the non-fixed SCO I limitation would not be exceeded.

c. The non-fixed (removable) accessible limits in this table have **NOT** been adjusted to reflect the default 10% swipe efficiency.

Low Toxicity Alpha Emitter Definition from 49 CFR 173.403

Low toxicity alpha emitters are: (1) Natural uranium, depleted uranium, and natural thorium; (2) Ores, concentrates, or tailings containing uranium-235, uranium-238, thorium-232, thorium-228 and thorium-230; or (3) Alpha emitters with a half-life of less than 10 days.

NUREG 1608 Interpretations

NUREG 1608 RAMREG 003⁵ is a joint guidance document prepared by NRC and DOT. It is not a regulation, but provides a great deal of guidance about methods and concepts for compliant categorization of SCO. For purpose of this paper, it is referred to simply as NUREG 1608. The following excerpts are especially pertinent, however the complete text of the guidance document should be consulted in order to assure compliance with the regulations.

NUREG 1608, Section 3.2.3 defines an accessible and inaccessible surface in the context of SCO shipments: "An accessible surface is any surface which can readily be wiped by hand, using standard radiation-measuring techniques. Any other surface is an inaccessible surface." This definition applies under normal non-accident conditions and represents what a person would encounter if the package were opened under normal conditions. If a 300-cm² area could be reached by a person's hand it is an accessible surface, otherwise it is not. Small openings are not accessible by this definition. It is generally good practice to seal off openings and the ends of pipes or tanks. This will minimize the likelihood that contamination will be released within the package due to rough handling during packaging, storage or shipment.

In NUREG 1608 section 3.1.4, the clarification is made that an object could be categorized as SCO, even if the surfaces were contaminated to both the limits for beta and gamma emitters and low toxicity alpha emitters, and to the limit for all other alpha emitters.

In NUREG 1608 section 3.1.6, guidance is provided that the most appropriate category should be used to assure that response to any incidents is suitable for the materials present. However, it is permissible to categorize radioactive material to higher categorization levels and package and ship them accordingly.

In NUREG 1608 section 3.1 "[DOT] regulations do not require measurements of contamination as the only means of accomplishing [SCO] determinations." Furthermore, "A reasoned argument could be used to categorize the great majority of candidate SCOs without the need for detailed quantitative measurements of fixed, accessible contamination, or total inaccessible contamination."

In NUREG 1608 section 6.2 guidance is provided for methods of rendering accessible surfaces inaccessible and fixing removable contamination.

Limited Quantity Definition from 49 CFR 173.403

“Limited quantity of Class 7 (radioactive) material means a quantity of Class 7 (radioactive) material not exceeding the materials package limits specified in Sec. 173.425 and conforming with requirements specified in Sec. 173.421.”

Section 173.421 describes the conditions under which exceptions to the specification packaging, marking, labeling, and certification can be taken.

Table 7 in Section 173.425 shows that Limited Quantity for solids is up to 10^{-3} times the A_2 quantity for a particular radionuclide or mixture of radionuclides. The A_2 quantity for weapons grade plutonium changes as a function of time. The calculated values for A_2 are shown in Table 4-2 of this document. These A_2 values are the same as the ones used in the WEMS database.

2.5 Conveyance Limit of 100 Times A_2 from 49 CFR 173.427

Table 9 of the regulation shows the “Conveyance Activity Limits for LSA Material and SCO.” Conveyances carrying SCO are limited to 100 times the A_2 value.

Nevada Test Site Waste Acceptance Criteria

The Nevada Test Site (NTS) is the principal recipient of low-level waste characterized using an SCO method. The NTS Waste Acceptance Criteria⁶ (WAC) specifies that all waste acceptable at NTS must be radioactive and meet certain criteria which are summarized below:

Radiological Characteristics

- ✓ The concentration of alpha-emitting transuranic nuclides with half-lives greater than 20 years must not exceed 100 nCi/g.
- ✓ The net weight of the waste, excluding the weight of the container and shielding, must be used to calculate the specific activity of the waste in each container.
- ✓ Sealed sources are evaluated individually against specific limitations.
- ✓ Waste equivalent to “Greater-than-Class C”⁷ is not acceptable

Accuracy and Uncertainty

- ✓ Characterization of waste shall be done with sufficient accuracy to permit proper segregation, treatment, storage, and disposal.
- ✓ When waste streams are characterized by sampling and analysis, the process shall be controlled and documented. The propagation of error throughout the sampling and analysis process shall be evaluated and considered when ascertaining usability of data for characterization of waste
- Package Restriction
 - ✓ Weight limits of 9000 pounds per box or 1200 pounds per 55-gallon drum apply. Special provisions for disposal of bulk waste or large items may be available. Void space in packaging must be minimized.
 - ✓ Large quantities of fine particulates shall be immobilized or in enclosed secure packaging.

Other Hazardous Constituents

- ✓ Hazardous waste is not acceptable. This includes compressed gasses, pathogens, reactive material, pyrophorics, and explosives.
- ✓ Limits on the concentration of PCBs and chelating agents apply.
- ✓ Waste containing asbestos is acceptable in some cases, subject to certain marking requirements, notification and segregation.
- ✓ Beryllium waste is acceptable if the container is appropriate and certain labels are applied.
- ✓ Lead may be added to a container for shielding purposes if it was not contaminated when it was added.

2.7 Envirocare Waste Acceptance Criteria

Envirocare of Utah, Inc may be used by RFETS for disposal of mixed waste characterized using an SCO method. Envirocare's WAC⁸ establishes these pertinent limits:

Prohibited Items

- Hazardous waste that is not also a radioactive waste
- Bulk liquid wastes, non-aqueous liquids, or wastes with an organic liquid phase
 - Water or air reactive wastes and materials
 - Pyrophoric wastes and materials
- DOT Forbidden, Class 1.1, Class 1.2 and Class 1.3 explosives
- Shock sensitive wastes and materials
- Batteries
- Compressed gas cylinders, unless they meet the definition of empty containers
- Mixed waste where the radioactive portion, at the time of disposal, will exceed the limits set forth in Envirocare's Radioactive Material License

- EPA waste codes F020, F021, F022, F023, F026, and F027; and Utah waste codes F999 and P999.

Activity Limits and Uncertainty Constraints

Envirocare's Radioactive Materials License (License # UT 2300249 Amendment #11) allows for disposal of special nuclear materials found in weapons grade plutonium at concentrations up to 10 nCi/g.^a Explicit in this limit is that measurement uncertainty at the one-sigma level cannot exceed 1.5 nCi/g. This measurement uncertainty limit does not include the uncertainty introduced by sampling error. In other words, 15% measurement uncertainty is allowable.

This level of measurement uncertainty is difficult to achieve with standard radiation safety procedures and instruments, which are calibrated to within $\pm 10\%$, and performance tested daily to within $\pm 20\%$. Instrument procedure modifications may be required to meet this criterion for disposal at Envirocare.

^a The limit for Pu-241 disposal is 350 nCi/g and the measurement uncertainty limit for this is 50 nCi/g. In practical terms, this higher value has little impact on the 10 nCi/g limit for a WG Pu mixture.

3. Statistical Methods for Plans A, B, and C

Two statistical methods, Plans A and B, and one non-statistical method, Plan C, have been developed for characterization of surface contaminated objects. Plan A is a non-parametric approach that was implemented in revisions 0 and 1 to RSP-09.05. The goal of Plan A is to conservatively estimate the amounts of activity in a group of surface contaminated objects and provide high confidence that applicable regulatory limits and waste acceptance criteria are met. In exchange for high confidence of compliance, this method produces a high potential for inappropriately failing surface contaminated objects that, in fact, conform to shipping and waste acceptance criteria.

Plan B is newly defined in this document. Plan B is a parametric approach that includes a test for non-normality of the sample data. After failing to reject the null hypothesis that the sample data are normally or lognormally distributed, the mean and upper tolerance limit (UTL) are calculated. The UTL is used for comparison to the applicable SCO limit. The mean is used to calculate package contents and specific activity. The result is a less conservative and more powerful statistical analysis that makes fewer inappropriate Type II errors.

Plan C, a non-statistical approach is also newly defined. In this simple case, where one or several surface contaminated objects can be completely measured, no statistical inference is required. Since the contamination levels are fully known, the measured maximum values shall be used for comparison to SCO limits and the mean value may be used in calculation of package contents specific activity.

Both Plans A and B require that candidate objects for SCO classification be grouped into populations with similar contamination levels. This grouping is based on location, knowledge of process, and professional judgement. After grouping has occurred, sampling and measurement is performed to obtain data that are used to infer population characteristics. Not all objects or surfaces in the population need to be sampled (measured), but all accessible areas must have an equal opportunity to be sampled. Inaccessible surfaces are assumed to be an unsampled subset represented by the sampled accessible surfaces. This approach for inaccessible surfaces is supported in NUREG-1608: “[DOT] regulations do not require measurements of contamination as the only means of accomplishing [SCO] determinations.” Furthermore, “A reasoned argument could be used to categorize the great majority of candidate SCOs without the need for detailed quantitative measurements of fixed, accessible contamination, or total inaccessible contamination.”

If contamination levels of inaccessible surfaces are not expected to be similar to accessible surfaces (which might occur on a tank or pipe interior or inside an air mover), then the inaccessible surfaces must be measured using a different, reliable method, or the inaccessible surface must be made accessible for measurement (cut open), or the object must be removed from the population.

3.1 Plan A, Non-Parametric Method

Plan A is a simple and conservative non-parametric statistical method. The goal of Plan A is to estimate the maximum percent of objects that may exceed the applicable SCO limit with an acceptable level of confidence.

At least 30 randomly collected measurements of surface activity are gathered and individually evaluated. None of the 30+ samples may exceed the applicable SCO limit. Additionally, neither the median and standard deviation of the sample data may exceed $\frac{1}{2}$ the SCO limit. The median is the value, which demarks the middle of the data, i.e., half of the data is above and half is below. In a symmetric data set, the median and mean (average) coincide but in a skewed data set they diverge, with the mean being drawn more in the direction of the few extreme values.

The choice of minimum sample size of 30 is based in using the sign-test (described in MARSSIM ⁹). The collection of swipes from randomly selected locations ensures that the total data set will tend to be like the total surface area being evaluated with the degree of similarity between the sample and the represented population growing larger as the sample size increases.

Because non-parametric methods derive from general probabilistic considerations and are meant to apply regardless of the shape or quantitative characteristics of the actual underlying data distribution, the results obtained can often be improved by taking advantage of known parametric restrictions on the data and the phenomenon being evaluated. However, without resorting to the use of those restrictions, the following simple results can be obtained.

Whatever the underlying data distribution, if the tail probability for values above the SCO limit is some value P , then the level of confidence associated with a sample of size n which contains no observations exceeding the SCO limit is $1-(1-P)^n$ that the proportion of material in the population which has greater than the SCO limit of contamination does not exceed P . Table 3-1 shows the level of confidence associated with a sample size of $n = 30$ and the proportion of population which may exceed the SCO limit of contamination given that no observation in the sample exceeds the SCO limit. For example, the method provides 95.8% confidence that the proportion of material which could exceed the limit is no greater than 10%. However, there can be only 78.5% confidence that it is no greater than 5%, based solely on the non-parametric approach.

Table 3-1. Level of Confidence that Proportion is No Greater Than P% for Sample of Size 30, Given that No Observation Exceeds SCO Limit

Maximum Percent (P%) Exceeding SCO Limit	Confidence Level (%)
1	26.0
2	45.5
3	59.9
5	78.5
7	88.7
10	95.8
15	99.2
20	99.9

The additional restrictions that the median and the standard deviation of the observed data shall not exceed a value of one-half the applicable SCO limit may serve to strengthen confidence that only a small proportion, if any, of the material being sampled exceeds the SCO limit. The effect of restrictions of known or expected distributional parameters is that we are approximately 95% confidence that the proportion of contaminated surface area exceeding SCO limit would be less than 5%. The rationale for this conclusion is described below.

Very-low and near-background contamination data most often approximately follow a log-normal probability distribution¹⁰, while generally elevated contamination data may tend to be less right-skewed in distribution and may more closely approximate a normal distribution. Therefore the median and standard deviation restrictions must be evaluated both under an assumption of normality and under an assumption of log-normality.

3.1.1 Normally Distributed Variate

Under the assumption of normality, the mean and the median should be nearly coincident and equal. However, the constraints of median and standard deviation less than one-half SCO limit could only be approached by the median. Since the measured values are also constrained by zero on the left, no sample from a normal population could be expected to demonstrate a standard deviation of the same magnitude as the constraint defined for the median. The standard deviation of a sample from an underlying population which could be treated as approximately normally distributed while bounded by zero on the left could not exceed a magnitude of less than about one-half the mean. Otherwise, the sample would be too right-skewed to be adequately modeled by a normal distribution and would fail accepted tests for non-normality. Thus, for a sample with median (and mean) near one-half SCO limit and from an

approximately normally distributed population, the sample standard deviation would be at most about one-quarter of the SCO limit.

Modeling such a population shows that a sample of size 30 taken from a normally distributed population and having a sample mean of one-half SCO limit and sample standard deviation of one-quarter of the SCO limit would yield approximately 95% confidence that the proportion of contaminated surface area exceeding SCO limit would be less than 5%. This conclusion is derived as follows:

For computational convenience, we will set the SCO limit equal to 1 with appropriate units. This implies that both median and mean are less than or equal to one-half (0.5) and the standard deviation is no greater than about one-quarter (0.25) in the same units in order to meet the criteria. The limiting case exists when the sample mean does, in fact, equal one-half the SCO limit, and the sample standard deviation equals one-quarter of the SCO limit. If either value is less, the conclusion is stronger. Under the limiting conditions the probability that any single observation on X is less than the SCO limit is

$$\Pr(X < 1) = \Pr\left(Z = \frac{X - \mu}{\sigma} < \frac{1 - \mu}{\sigma} = \frac{1 - 0.5}{0.25} = 2\right)$$

where Z is the Standard Normal variate.

Because the observed mean, median, and standard deviation are estimates of the true parameters, the probability calculations must also be performed using the associated upper confidence limit values for the estimates of mean and standard deviation. This will serve to put bounds on the achievable levels of confidence for the likely estimates of proportions of material, which may exceed the SCO limit under the given conditions. Since the mean and standard deviation of deposited surface contamination often exhibit some degree of positive correlation, the effect would tend to liberalize the confidence level estimates. Therefore, the more conservative choice could be made when computing confidence to offset this influence. However, the confidence interval estimates for the limiting case are largely independent of each other, so the combined influence may be treated as multiplicative. Thus, use of upper 80% confidence limits for the two parameter estimates should yield a combined upper confidence level of about 96% ($1 - 0.2^2$) or more that the proportion exceeding the SCO level is less than the calculated value.

For relatively large ($n > 29$) samples the upper $(1-\alpha)100\%$ confidence limit for the population mean is found by¹¹

$$UCL_{(1-\alpha)100,(\mu)} = \bar{x} + t_{\alpha,n-1} \frac{s}{\sqrt{n}}$$

For sample size $n = 30$; observed mean \bar{x} equal to 0.5; observed standard deviation s equal to 0.25; and $\alpha = 0.2$, this equation yields UCL_{80} for μ as follows:

$$UCL_{80,(\mu)} = 0.5 + 0.8542 \cdot (0.25/\sqrt{30}) = 0.5390$$

In addition, the upper confidence limit for standard deviation is found from

$$UCL_{(1-\alpha)100,(\sigma)} = \sqrt{(n-1)s^2/\chi_{\alpha,n-1}^2} = s\sqrt{(n-1)/\chi_{\alpha,n-1}^2}$$

Under the same conditions, this yields:

$$UCL_{80,(\sigma)} = 0.25 \cdot \sqrt{29/22.475} = 0.2840$$

Solving the earlier equation for the probability that any one X exceeds the defined SCO Limit of 1.0 while using these conservative estimates for mean and standard deviation yields

$$\Pr(X < 1) = \Pr\left(Z = \frac{X - \mu}{\sigma} < \frac{1 - \mu}{\sigma} = \frac{1 - 0.5390}{0.2840} = 1.623\right) = \Pr(Z < 1.623) = 0.9477 \approx 95\%$$

Assuming relative independence of the two UCL parameter values, this result implies we can be at least 95% confident that the proportion of the population exceeding the SCO limit is less than 5%.

3.1.2 Lognormally Distributed Variate

If the variate L has a Lognormal probability distribution, then $\log_n L$ (or $\ln(L)$) has a Normal probability distribution with mean μ and standard deviation σ with the following relationships between the Lognormal and Normal distributions¹²

$$\text{Median of } L, \quad m = e^{\mu} \quad \text{or} \quad \mu = \ln(m)$$

$$\text{Mean of } L, \quad \Lambda = m \cdot e^{\sigma^2/2}$$

$$\text{Variance of } L, \quad \zeta^2 = m^2 \cdot e^{\sigma^2} (e^{\sigma^2} - 1)$$

For the restrictions that the median and standard deviation of L both be no greater than one-half the SCO limit, and by defining the SCO Limit in this case as 2.0 with appropriate units, the mean of $\ln(L)$ is $\ln(1)$ or $\mu = 0$ and the standard deviation of $\ln(L)$ is $\sigma = 0.69369$. The transformed SCO limit is $\ln(2) = 0.693147$.

As before, under these conditions the probability that any single observation on L is less than the SCO limit is

$$\Pr(L < 0.693147) = \Pr\left(\frac{0.693147 - \mu}{\sigma} = \frac{0.693147 - 0}{0.69369}\right) \\ = \Pr(Z < 0.9992) = 0.8412$$

where Z is the Standard Normal variate.

This result is similar to that for the normal variate. Applying the UCL_{80} formulas for normal variates to the log-transformed data yields:

$$UCL_{80,(\mu)} = 0.0 + 0.8542 \cdot (0.69369 / \sqrt{30}) = 0.10818 \\ \text{and } UCL_{80,(\sigma)} = 0.69369 \cdot \sqrt{29/22.475} = 0.78798$$

Solving the earlier equation for the probability that any one L exceeds the defined SCO Limit of 0.693147 while using these conservative estimates for mean and standard deviation yields

For $UCL_{80,(\mu)}$ & $UCL_{80,(\sigma)}$:

$$\Pr(L < 0.693147) = \Pr\left(\frac{0.693147 - \mu}{\sigma} = \frac{0.693147 - 0.10818}{0.78798}\right) \\ = \Pr(Z < 0.7424) = 0.7711$$

Assuming relative independence of the two UCL values, this result implies we can be 95% confident that the proportion of the population exceeding the SCO Limit is less than 22.89%

Again, this result is similar to, but less restrictive than, that obtained for a normally distributed variate. For log-normally distributed variates, the theoretical results indicate that the non-parametric approach of the first section yields greater confidence and/or lower proportions of material potentially exceeding the SCO limit.

3.2 Plan B, Parametric Method

In application, Plan A, the non-parametric approach, may prove to be unduly conservative. Plan B takes advantage of the observed data distributional features to generate more definitive quantitative characterization results.

Characterization of the magnitude of material contamination may be based upon two separate but related criteria; (1) the greatest value the average (mean) concentration might be reasonably expected to have, and (2) the largest value some randomly selected sample might reasonably be expected to display.

The first criterion is applied by using the observed characteristics of the sample data to mathematically model the distribution of likely mean values for the underlying population of all possible sample values and determine the largest value that the true mean would be expected to be, with some level of confidence. This is usually performed by assembling a moderately large sample of randomly selected observations on the quantity and applying the logic of the following paragraphs.

The second criterion is applied by calculating a particular upper tolerance limit (UTL) value for which there is a 90% confidence level that at least 98% of the surfaces in the sampled population will be less than the calculated value. The UTL is calculated using a method described in "Calculating One-Sided Limits Based on Weighted Means from Multiple Samples" by Palachek.¹³ The equations used are somewhat simplified because there is no need in this application to use the features in the equations that accommodate weighted means from multiple samples.

3.2.1 Calculation of UTL

Calculation of the UTL is based on equation 8 in Palacheck's paper.

$$UTL = Y_w + kS_p \quad (1)$$

where Y_w is a weighted mean, but in this case is simply the mean
 S_p is the pooled sample standard deviation, and

$$k = \frac{t'_{n-1, 1-\alpha, (\sqrt{n}Z_p)}}{\sqrt{n}} \quad (\text{Palachek's equation 19})$$

A value, t' , from the noncentral t distribution calculated using Palachek's equation 23, is used to compute k.

$$t'_{\nu, \alpha}(\delta) = \frac{\delta + Z_{1-\alpha} \sqrt{1 + \frac{(\delta^2 - Z_{1-\alpha}^2)}{2\nu}}}{1 - \frac{Z_{1-\alpha}^2}{2\nu}}$$

where

ν is the degrees of freedom. In this case $\nu = n - 1$

Z_p is the Pth percentile from the standard normal distribution

n is the number of samples

δ is the non-centrality parameter. In this case $\delta = Z_p \times n^{1/2}$

Both criteria (the mean and UTL) are valid only when certain assumptions are met. Specifically, for the mean criteria the sample data must either have been drawn from an underlying population having a normal (Gaussian) probability distribution or the sample size must be large enough to be able to validly apply the Central Limit Theorem (CLT), also known as the Normality Convergence Theorem, and for the UTL criteria the sample data must have been drawn from an underlying population having a normal (Gaussian) probability distribution.

3.2.2 Test for Normality

The normality of the underlying distribution can be mathematically tested using one of several statistical tests. We have chosen to use the D'Agostino Omnibus test, which considers both skewness and kurtosis. This test is considered increasingly valid for sample sizes larger than about 20, but is adequate for sample sizes as small as 15. Sample size less than 20 is justified, as we are most concerned with generating a good estimate for the mean and kurtosis has less impact for this application than skewness. Kurtosis is not concerned with the degree of non-symmetry, only the degree of spread. Data spread has no direct influence on the arithmetic mean. Smaller sample sizes could be used if an alternative test for non-normality, such as the Shapiro-Wilk test, was chosen.

R. B. D'Agostino, et al,¹⁴ describe a test for non-normality that combines statistics from tests for both skewness and kurtosis. These two statistics are z_s and z_k which are calculated as described later. The Omnibus test statistic, K^2 , is approximately distributed as a chi-square with two degrees of freedom and is calculated as:

$$K^2 = z_s^2 + z_k^2$$

Thus, if K^2 is greater than 5.991, the data are significantly non-normal at $\alpha = 0.05$. That is, there is less than 5% chance that the data is from a normal (Gaussian) distribution.

3.2.3 Calculation of D'Agostino Skewness Test Statistic, z_s

Because the normal distribution is symmetrical, the skewness coefficient, $\sqrt{b_1}$, is equal to zero for normal data. D'Agostino describes a test to determine if the value of $\sqrt{b_1}$ is significantly different from zero. The skewness statistic, z_s , is, under the null hypothesis of data normality, itself approximately normally distributed. This statistic, which is restricted to sample sizes $n > 8$, is computed as:

$$z_s = d \ln \left(\frac{T}{a} + \sqrt{\left(\frac{T}{a} \right)^2 + 1} \right)$$

where

$$d = \frac{1}{\sqrt{\ln(W)}}$$

$$W^2 = -1 + \sqrt{2(C-1)}$$

$$C = \frac{3(n^2 + 27n - 70)(n+1)(n+3)}{(n-2)(n+5)(n+7)(n+9)}$$

$$T = \sqrt{b_1 \left(\frac{(n+1)(n+3)}{6(n-2)} \right)}$$

$$b_1 = \frac{m_3^2}{m_2^3}$$

$$m_k = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^k$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

$$a = \sqrt{\frac{2}{W^2 - 1}}$$

3.2.4 Calculation of D'Agostino Kurtosis Test Statistic, z_k

In the normal distribution, the theoretical value of the kurtosis coefficient, b_2 , is equal to three. D'Agostino describes a test to determine if the value of b_2 is significantly different from three. The statistic, z_k , is, under the null hypothesis of normality, approximately normally distributed. This statistic, which is an approximation for sample sizes $n < 20$, is computed as:

$$z_k = \frac{\left(1 - \frac{2}{9A}\right) - \left(\frac{1 - \frac{2}{A}}{1 + G \sqrt{\frac{2}{A-4}}} \right)^{\frac{1}{3}}}{\sqrt{\frac{2}{A-4}}}$$

where

$$A = 6 + \frac{8}{E} \left(\frac{2}{E} + \sqrt{1 + \frac{4}{E^2}} \right)$$

$$E = \frac{6(n^2 - 5n + 2)}{(n+7)(n+9)} \sqrt{\frac{6(n+3)(n+5)}{n(n-2)(n-3)}}$$

$$G = \frac{b_2 - \left(\frac{1}{n+1} \right)}{\sqrt{\frac{24n(n-2)(n-3)}{(n+1)^2(n+3)(n+5)}}}$$

$$b_2 = \frac{m_4}{m_2^2}$$

$$m_k = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^k$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

3.2.5 Calculation of Mean and UTL for Log-normal Data

If the detected contamination concentrations exhibit a highly “right-skew” characteristic, that is, most observed values are very small or near zero with a few data values scattered at higher levels with diminishing frequency, near normality can often be achieved by taking the natural logarithm of the data values. Right-skew data that can be translated to normality in distribution by performing a log-transformation are said to follow a log-normal (LN) distribution. If the log-transformed sample data-set passes the D’Agostino Omnibus test, the data may be treated as log-normal and the upper tolerance limit (UTL) and other quantities of interest calculated.

Due to the lack of symmetry, right-skewed data distributions have means that are always larger than the distribution median. The log-transformation to achieve normality brings the mean and median into closer coincidence for the transformed data but the reverse transformation (exponentiation) separates the mean and median again. Only order statistics, such as the minimum, maximum, median and UTL can be transformed and back-transformed directly. The median ($Median_{(LN \text{ data})}$) of the underlying log-normal data is obtained directly by exponentiation of the mean ($\mu_{(LN-Xform)}$) of the log transformed data using Equation (2).

$$Median_{(LN \text{ data})} = e^{\mu_{(LN-Xform)}} \quad (2)$$

Although the mean ($Mean_{(LN\ data)}$) of the underlying log-normal data is necessarily larger than the median, the statistical literature shows that it can be calculated from the mean and variance ($\sigma^2_{(Ln-Xform)}$) of the log-transformed data using Equation (3).

$$Mean_{(LN\ data)} = e^{\mu_{(Ln-Xform)} + \frac{1}{2}\sigma^2_{(Ln-Xform)}} \quad (3)$$

The upper tolerance limit ($UTL_{(LN\ data)}$) of the underlying log-normal data is obtained directly by exponentiation of the upper tolerance limit ($UTL_{(Ln-Xform)}$) of the log-transformed data using Equation (4).

$$UTL_{(LNdata)} = e^{UTL_{(LN-Xform)}} \quad (4)$$

$UTL_{(LN-Xform)}$ is obtained using Equation (1) with parameters Y_w and S_p calculated from the log-transformed data.

4. Radionuclides and Nuclear Data

4.1 Plutonium

If specific data for the activity of each radionuclide is available, that should be used. Data of this extent are seldom available, and conservative assumptions incorporated into scaling factors are used for most calculations. The site assumes, for calculation purposes, that the weapons grade (WG) plutonium mixture is composed of the material shown in Table 4-1 on the date when it was chemically separated to extract americium (termed the strike date).

Table 4-1. Assumed Weight Percent of Weapons Grade Plutonium at the Time of Chemical Separation, and Other Constants Used in Calculations.

Radionuclide	weight percent	Mode of Decay	Half-life, y	Isotopic Specific Activity of Pure Radionuclide, Ci/g
Pu-238	0.00993%	Alpha	87.7	17.1
Pu-239	93.5%	Alpha	24,100	0.0621
Pu-240	5.73%	Alpha	6560	0.227
Pu-241	0.359%	Beta	14.4	103
Pu-242	0.0305%	Alpha	3.75×10^5	0.00394
Am-241	0.0175%	Alpha	433	3.43

To calculate specific activity of the WG Pu, an assumption regarding the age of the plutonium, or the strike date is required. In 2001, the assumption is made that the strike date for WG Pu was 32 years ago. When the mixture in Table 4-1 is decayed to the appropriate date, the specific activity and A_2 value shown in Table 4-2 result. Table 4-3 shows the activity distribution among the WG Pu isotopes as a function of time. These are the same values used in the RFETS WEMS database. Uncertainty in the assumed strike date is substantial. Therefore it is not appropriate to calculate the activity ratios using any date that is more refined than the nominal year. Uncertainty due to these assumptions is discussed in Section 7.4

Table 4-2. Specific Activity and A₂ Value of WG Pu as a function of date.

Calendar Year	TRU Alpha Activity, Ci/g	Beta Activity, Ci/g	Total ($\alpha + \beta$) Activity, Ci/g	A ₂ Value, Ci
1989	0.0802	0.1483	0.228	0.0149
2000	0.0821	0.0832	0.165	0.0107
2001	0.0822	0.0793	0.161	0.0104
2002	0.0823	0.0756	0.158	0.0102
2003	0.0823	0.0720	0.154	0.0100
2004	0.0824	0.0686	0.151	0.00975
2005	0.0825	0.0654	0.148	0.00955
2006	0.0826	0.0623	0.145	0.00935
2007	0.0826	0.0594	0.142	0.00917
2008	0.0827	0.0566	0.139	0.00899
2009	0.0828	0.0540	0.137	0.00882
2010	0.0828	0.0514	0.134	0.00866

Table 4-3. Assumed Activity Ratios for WG Plutonium as a Function of Date.

Calendar Year	Activity Percent Pu-238	Activity Percent Pu-239	Activity Percent Pu-240	Activity Percent Pu-241	Activity Percent Pu-242	Activity Percent Am-241
1989	1%	25%	6%	65%	0.001%	3%
2000	1%	35%	8%	50%	0.001%	6%
2001	1%	36%	8%	49%	0.001%	6%
2002	1%	37%	8%	48%	0.001%	6%
2003	1%	38%	8%	47%	0.001%	7%
2004	1%	38%	9%	45%	0.001%	7%
2005	1%	39%	9%	44%	0.001%	7%
2006	1%	40%	9%	43%	0.001%	7%
2007	1%	41%	9%	42%	0.001%	7%
2008	1%	42%	9%	41%	0.001%	8%
2009	1%	42%	9%	39%	0.001%	8%
2010	1%	43%	10%	38%	0.001%	8%

4.2 Uranium

Data for uranium are found in Radiological Engineering Technical Basis Document 00146, Calculation of Beta Activity for Depleted Uranium, Highly Enriched Uranium and Aged Weapons Grade Plutonium.¹⁵

5. Surface Contamination Measurement Methods

5.1 *Instrumentation and Calibration Quality*

Only measurement systems capable of providing quality-assured data and approved for use at RFETS shall be used to obtain data needed to characterize surface contaminated objects.

Any radiological measurement data used to characterize SCO shall be obtained with instruments that have NIST (or other national or international standard) traceability. If no traceability to national or international standards is available, as may be the case for surface contamination standards approaching the SCO II limits, the best available technology shall be applied.

For measurement of contamination levels within the range of standard survey meters (0 to 1.2 million dpm/100 cm²) the requirements of ANSI N323¹⁶ shall apply. Calibration uncertainty shall be no greater than $\pm 10\%$. Daily performance testing of the instrumentation shall limit instrument uncertainty to less than $\pm 20\%$.

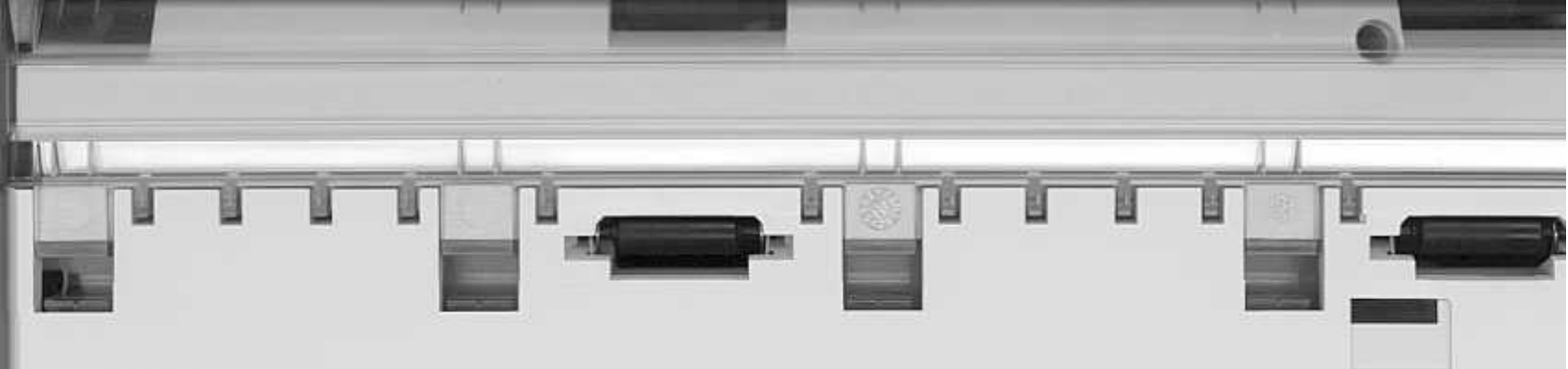
5.2 *Wiping efficiency*

Wiping efficiency, which is discussed in NUREG 1608, is normally assumed to be a default value of 10%. However, it may be useful to calculate wiping efficiency for a specific case or a group of items. The practice is specifically allowed for SCO determinations provided those efficiencies are documented and accounted for in contamination determinations.¹⁷ There is no mandatory standard method for determining wiping efficiency.^b However, ISO 7503-1 (1988)¹⁸ does provide a method for determination of a removal factor. If wiping efficiency, or removal factor, is empirically determined, the following method based on ISO 7503-1 should be followed for each important combination of contaminant and surface material:

On a surface where readily detectable amounts of non-fixed contamination exists, take sequential wipes on exactly the same area, using - to the extent possible - identical technique, until exhaustive removal is accomplished. In other words, continue to take sequential wipes until a wipe picks up only statistically negligible activity.

To calculate the removal factor, divide the activity on the first wipe by the sum of activity on all wipes as shown in the following equation.

^b Based on personal conversations between Robert Morris with Wendell Carriker of DOT and between Robert Morris with Rick Boyle of DOT. Both conversations were held on July 5, 2000.



$$removalFactor = W_1 / W_T$$

where

W_1 is the net activity removed in the first wipe, and

W_T is the net activity removed all subsequent wipes, including the first.

In practical terms, statistically negligible difference for this sequence of wipes would be reached when a subsequent wipe showed only 5% to 10% of the activity collected on the first wipe. For example, if the true removal factor was 20% and the sequence of wipes was stopped after a subsequent wipe contained only 10% of the activity found in the first wipe, the removal factor would be estimated to be 22%. This difference from the true value of 20% is judged to be insignificant for the use described here.

Non-fixed contamination levels are determined by dividing the activity removed in a wipe by the appropriate removal factor.

In some cases when operating near instrument detection limits, the wiping efficiency-corrected non-fixed level exceeds the fixed plus non-fixed contamination level measured prior taking to the wipe at that location. This illogical outcome stems from the error propagated by multiplying two poorly defined values. When this occurs, the fixed plus non-fixed contamination value (i.e., the direct measurement of total contamination) should be substituted for the non-fixed contamination value. In other words, if the removable contamination level exceeds the total contamination level, the removable contamination value is discarded and replaced with the total value and then all of that contamination is assumed to be removable.

6. Calculation of Shipping Container Contents

6.1 Surface Area to Mass Ratios

The SCO characterization method establishes either a conservative (UCL95) estimate of the surface contamination level on the objects of interest (Plan A) or an unbiased estimate (mean) of the contamination level (Plans B and C). The contamination levels are in units of activity (dpm) per unit area (cm²). To determine the total activity or activity concentration in a shipping container the surface area of the contaminated objects in the container must be known. Surface area can be directly measured; however, the direct measurement method may, in some cases, be difficult or error-prone. An alternate method is to weigh the items of interest, then multiply the weight by a predetermined Surface Area to Mass Ratio that is representative of the material. Surface Area to Mass Ratios, based on measured or tabulated data, have been established for a wide variety of materials and objects. Table 6-1 shows the Surface Area to Mass Ratio for several materials. Appendix B documents some empirical measurements used to create Table 6-1. For sheet materials (materials in which the edge area is negligible) not covered here, the ratio may be approximately derived using tabulated handbook data and the following equation for objects contaminated on one surface only:

$$SAtoMassRatio_1 = \frac{1}{(Thickness \times density)}$$

For objects contaminated on both surfaces (in other words, both an inside and an outside), the calculation is made using the following equation.

$$SAtoMassRatio_2 = 2 \times \frac{1}{(Thickness \times density)}$$

In some cases, materials may be intimately mixed (commingled) and it may be difficult to determine what ratio to apply. RF/RMRS-98-242 documented a survey of the materials identified for disposal in Building 779. This work suggests that waste materials can be conservatively characterized by assuming that all of the waste is 22-gauge sheet metal. Subsequent experience with SCO waste has shown that this is usually, though not always, a conservative assumption.

For Plan A evaluations only, the Surface Area to Mass Ratio may be accurately estimated or may be estimated using one of three default values shown in Table 6.2. In most cases, Plan A evaluations are performed when the surface contamination levels are relatively low and the applicable SCO limits and waste acceptance criteria limits are not challenged. Consequently, the error induced by use of default values will not typically result in total or specific activity differences that are important to waste disposal facilities.

The default values in Table 6-2 were selected to conservatively represent the Surface Area to Mass Ratio of waste categories without grossly overestimating the value. Paper and plastic sheeting are represented by the value for 0.01" plastic. Uncertainty studies described in Section 7.3 guided the selection of these default values. Plywood, rubber, cardboard, and thick plastic (similar to tupperware or thicker) is a medium category. All other objects, including trivial amounts from the other categories, may use the lowest value. Trivial amounts are assumed to be less than about 3% of the net weight of the container.

When default assumptions are applied, the uncertainty in the calculation of shipping container activity and activity concentration is significantly affected by the default value chosen. Therefore, any use of a default Surface Area to Mass Ratio must be done with caution and knowledge of the waste stream.

Table 6-1. Surface Area to Mass Ratios for Various Materials.*

Type of Material	Sheet Thickness (inches)	One side contaminated (cm ² /g)	Two sides contaminated (cm ² /g)
Aluminum	0.125	1.19	2.38
Asbestos – Transite Panels	0.125	1.28	2.57
Asphalt – Pavement	4	0.04	0.09
Benelux (ARBORON®)	2	0.14	0.28
Bucket, plastic			9.2
Carbon Steel – Rolled	0.25	0.20	0.40
Carbon Steel	0.5	0.10	0.20
Carbon Steel – Rolled	1	0.05	0.10
Cardboard box			42
Concrete – 4" Slab	4	0.04	0.08
Lead – Rolled	0.25	0.14	0.28
Lead – Rolled	0.5	0.07	0.14
Lead – Rolled	0.75	0.05	0.09
Lead – Rolled	1	0.03	0.07
Lead glass - 0.3125" (5/16")	0.3125	0.6	1.2
Paper	0.01	32.74	65.48
Personal computer case and contents			0.56
Personal computer monitor			0.40
Polyethylene Sheeting (Plastic sheet - 10 mil)	0.01	42.56	85.11
Polyethylene Sheeting (Plastic sheet - 20 mil)	0.02	21.28	42.56
Polyethylene Sheeting (Plastic sheet - 40 mil)	0.04	10.64	21.28
Polyethylene Sheeting (Plastic sheet - 80 mil)	0.08	5.32	10.64
Plate Glass	0.25	0.61	1.22
Plate Glass	0.50	0.31	0.61
Plate Glass	0.75	0.20	0.41
Plate Glass	1	0.15	0.31
Plexiglas®	0.25	1.31	2.62
Plexiglas®	0.50	0.66	1.31
Plexiglas®	0.75	0.44	0.87
Plexiglas®	1	0.33	0.66
Plywood	0.50	1.28	2.56
Plywood	0.75	0.85	1.71
Printed circuit board			3.17
Rubber Goods	0.125	2.07	4.14
Sheet metal - 18 gage	0.048	1.03	2.07
Sheet metal - 20 gage	0.036	1.38	2.76
Sheet metal - 22 gage	0.030	1.66	3.32

Table 6-1. Surface Area to Mass Ratios for Various Materials (continued)*

Type of Material	Sheet Thickness (inches)	One side contaminated (cm ² /g)	Two sides contaminated (cm ² /g)
Sheetrock – gypsum	0.50	0.93	1.86
Stainless Steel	0.25	0.20	0.39
Stainless Steel	0.5	0.10	0.20
Stainless Steel	0.75	0.07	0.13
Stainless Steel	1	0.05	0.10
Wood Pine	0.25	3.78	7.56
Wood Pine	0.5	1.89	3.78
Wood Pine	0.75	1.26	2.52
Wood Pine	1	0.94	1.89
Wood, pine 2 x 4			1.70
Wood, pine 2 x 6			1.68

* Unless otherwise stated, the data used to calculate the values in the above table are determined based on the information contained in Marks Standard Handbook for Mechanical Engineers¹⁹; Pocket Reference.²⁰ by T. J. Glover.

Table 6-2. Default Surface Area to Mass Ratios for Use in SCO Plan A Evaluations

Type of Material	Surface Area to Mass Ratio (cm ² /g)
Plastic sheeting, paper	85
Plywood, rubber, cardboard, thick plastic	10
Structural metal and sheet metal, all other materials, including dimensional lumber and trivial [#] amounts of materials from other default groups	3

[#] Trivial amounts are assumed to be less than 3% of the package weight.

6.2 Calculation of Package Activity and Activity Concentration

To estimate the activity in a package, the surface area of all of the objects from the same Characterization Survey Unit (SCU) packed in a container is estimated, either from direct measurement or by application of a Surface Area to Mass ratio. The surface area is multiplied by an estimator of the fixed plus non-fixed contamination level for the SCU. For SCUs evaluated using Plan A, the UCL95 is used as the contamination level estimator. For SCUs evaluated using Plan B or Plan C, the mean is used as the estimator. This results in a total activity in the package attributable to objects from that SCU.

If a container is packed with objects from more than one SCU, a mass-weighted average is used to establish the total and specific activity in the package.

The activity concentration in the package is estimated by dividing the total activity in the package by the net weight of the surface contaminated objects. Dunnage, such as initially uncontaminated blocking and bracing material or incidental quantities of oil-dry, shall not be included in the net weight.

7. Uncertainty Evaluation

Evaluation and consideration of uncertainty is a requirement typically found in waste acceptance criteria. Uncertainty derives from many different sources during the characterization of waste. Major sources of uncertainty are described here using terms and methods defined in NIST Technical Note 1297.

Uncertainty is not used to bias the reported values developed from data. Instead it is used to judge the adequacy of the data for its intended purposes. The numbers cited here are intended for illustration and as a rough estimate leading to combined standard uncertainty. They should not be used beyond that purpose, because in some cases the component uncertainty values are based only on observation and professional judgement.

Components of uncertainty may be categorized according to the method used to evaluate them.

- Type A method of evaluation of uncertainty by the statistical analysis of series of observations,
- Type B method of evaluation of uncertainty by means other than the statistical analysis of series of observations.

Each component of uncertainty, however evaluated, is represented by an estimated standard deviation, termed standard uncertainty, equal to the positive square root of the estimated variance.

Type A standard uncertainty is obtained by statistically estimating standard deviation. Type B standard uncertainty component is obtained by estimating a quantity which may be considered an approximation to the corresponding standard deviation; it is equal to the square root of the corresponding variance and is obtained from an assumed probability distribution based on all the available information. When standard uncertainty is reported as a percentage, it becomes relative uncertainty.

7.1 Combined Relative Uncertainty

The combined relative uncertainty is estimated by a method typically called the “root-sum-of-the-squares.” Values for the various uncertainty terms detailed in Sections 7.2 through 7.5 are included in a sum of squares calculation for a best, typical, and worst case. The results, which are rounded to avoid the appearance that these values are well known, shown in Table 7-1.

Table 7-1. Assumed Combined Relative Uncertainty for SCO Evaluations.

Evaluation Method	Best Case Uncertainty	Typical Case Uncertainty	Worst Case Uncertainty
Plan A	40%	120%	400%
Plan B	30%	40%	90%
Plan C	30%	40%	60%

The wide uncertainty band for Plan A evaluations reflects the tradeoff inherent when conservative estimates are made as a way of ensuring that important regulator and waste acceptance limitations are not exceeded.

Radiological Measurement Uncertainty

Radiological measurement uncertainty includes both Type A and Type B components. The value of the uncertainty varies with instrument type. Estimates of uncertainty in calibration have been reported for many RFETS instruments. Table 7-2 shows values for a best, typical and worst case uncertainty assumption for use of SCO instruments. Typical and worst case uncertainty estimates include the possibility of ambient temperature effects (the Ludlum Model 12-1A is especially prone to this) and of minor technique errors by the operator. Therefore the best case scenario is simply a Type A estimation, where the typical and worst case estimate scenarios combine Type A and B uncertainty components.

Table 7-2. Assumed Uncertainty for Radiological Instrumentation Used in SCO Evaluations.

Evaluation Method	Best Case Uncertainty	Typical Case Uncertainty	Worst Case Uncertainty
Plan A, B and C	15%	25%	40%

Surface Area Estimation Uncertainty

Uncertainty in surface area estimation results directly in uncertainty of the total activity in the waste package in the uncertainty in the specific activity of the package. Surface area may be estimated using direct linear measurement, or by multiplying the measured mass by a Surface Area to Mass Ratio. For Plan A, the surface area is

normally estimated based on Surface Area to Mass Ratios. For Plans B and C, it is more likely that the area will be measured or very accurately estimated. Uncertainties in either the measurement of mass or of linear dimensions are a Type A component, with little, perhaps 5% uncertainty, and more typically about 30% uncertainty.

Significantly more uncertainty exists when surface area is estimated using measured weight multiplied by a Surface Area to Mass Ratio. The default values shown in Table 6-2 can be compared to values in Table 6-1. If the default for plastic sheeting and paper (85 cm²/g) is applied to plastic sheeting that is 0.08 inches thick (10.64 cm²/g), the error is 700%. For 0.04" plastic sheeting the error is 300% and for paper the error is 30%.

If the default for plywood, rubber cardboard, thick plastic, and thin sheet metal (10 cm²/g) is applied to 0.5" wood ((3.78 cm²/g), an overestimation error of 165% occurs. Cardboard (9.14 cm²/g) results in an overestimate of 9%.

Using the default value of 3 cm²/g for 0.25" steel results in an overestimation error of 670%. For 0.25" plexiglass the value is 15%. In practice the best estimate is likely to be about 2 cm²/g^c.

The Type B uncertainty overwhelms the Type A component in this evaluation, and therefor the Type A uncertainty is neglected. The assumptions for Type B component of uncertainty for best, typical, and worse cases are shown in Table 7-2. It must be emphasized that this estimate is based only on observation and professional judgement.

Table 7-3. Assumed Uncertainty for Surface Area Estimation for SCO Evaluations.

Evaluation Method	Best Case Uncertainty	Typical Case Uncertainty	Worst Case Uncertainty
Plan A	10%	100%	400%
Plan B	5%	20%	50%
Plan C	5%	20%	25%

7.4 Statistical Reliability of Estimates

^c Personal observation by Robert Morris based on container inventories from Building 707 during April, 2000 through March 2001. Building 707 had no large capacity scale, so inventories were taken in approximately 50 lb increments or less and the SA/M ratio was calculated on that basis.

For parametric analysis, such as Plan B, uncertainty in the reliability of the statistical estimate is a function of several factors including:

- sample size
- deviation of the population from normality
- selected confidence level
- mean activity of the population.

When a sample size of 30 or more is used, the estimate is only marginally improved by more sampling. As the population deviates from normality the uncertainty increases. When a high confidence level is selected, the range defining the confidence interval about the mean also expands, resulting in a larger uncertainty measure, but greater certainty that the parameter has been captured. When mean activity is low the relative standard deviation (standard deviation divided by the mean) tends to be higher, decreasing the precision of parameter estimates and increasing the degree of uncertainty.

Each of these variables has a wide range of possible values, making estimation difficult. For a sample size of 20 and fixed plus non-fixed activity levels of approximately 10,000 dpm/100 cm², the Type B uncertainty due to statistical reliability estimated for a best, typical and worst case is shown in Table 7-3. These estimates are based largely on observation and professional judgement.

Similar considerations apply to a non-parametric analysis, such as Plan A. However, the uncertainty is even greater as there is no inherent assumption of normality and the contamination level is typically low. The estimated values for Plan A are shown in Table 7-4. These estimates are based largely on observation and professional judgement.

Table 7-4. Assumed Uncertainty for Statistical Reliability of Estimates for SCO Evaluations.

Evaluation Method	Best Case Uncertainty	Typical Case Uncertainty	Worst Case Uncertainty
Plan A	30%	50%	80%
Plan B	10%	20%	50%
Plan C ^Φ	NA	NA	NA

Φ When NA (not applicable) is shown, a value of zero should be used for uncertainty calculations.

7.5 Uncertainty Due to Plutonium Separation Date

Assumptions regarding the plutonium separation (strike) date are discussed in Section 4.1. If the strike date were 1989, the last year of production, instead of the conservatively assumed date of 1969, the values used in scaling factors would be altered. Table 7-5 compares pertinent activity data, based on 1989 and 2001 strike dates.

Table 7-5. Comparison of Specific Activity and A_2 Value of WG Pu Separated in 1989 and 1969.

Year of Pu/Am Separation	TRU Alpha Activity, Ci/g	Beta Activity, Ci/g	Total ($\alpha + \beta$) Activity, Ci/g	A_2 Value, Ci
1989	0.0802	0.1483	0.228	0.0149
1969	0.0822	0.0793	0.161	0.0104
Percent difference based on 1969	2.3%	-87%	-42%	-43%

The difference in TRU alpha activity between the two dates, 2%, is negligible. TRU alpha activity is used for comparison to SCO limits and in determining acceptability for disposal in a near-surface landfill. The differences in total activity concentration and in the A_2 value are approximately -40%, and are closely correlated. The total activity in a package is compared to the A_2 value for compliance with SCO shipping regulations. Because of their correlation, this uncertainty has little or no effect on quantities important to compliance with SCO shipping regulations. If the plutonium had been more recently separated, the total activity in the package would be underestimated by about 40%. This would result in underestimation of the activity buried at the waste disposal site.

Considering these facts, the component uncertainty based on plutonium strike date is summarized in Table 7-6.

Table 7-6. Uncertainty Based WG Pu Separation Date for SCO Evaluations.

Evaluation Method	Best Case Uncertainty	Typical Case Uncertainty	Worst Case Uncertainty
Plan A, B, and C	5%	15%	30%

8. REFERENCES

PRO-267-RSP-09.05. "Radiological Characterization for Surface Contaminated Objects." Rocky Flats Environmental Test Site. Radiological Safety Practices Manual. 2001.

RF/RMRS-98-242, Rev. 0. Technical Basis Document, "Radiological Field Characterization of Low Level Waste by Measuring Surface Contamination and Calculating Total and Specific Activity." Rocky Flats Environmental Technology Site. June 15, 1998.

RFETS Interoffice Correspondence from Thomas R. Gatcliffe to Gary J. Bracken, dated November 24, 1998. Subject: "Statistical Analysis of Proposed Sample Data Restriction for Classification of Surface Contamination Objects (SCO) in RFETS Draft Procedure PRO-267-RSP 09.05 (Rev 1)." TRG-036-98.

RFETS Interoffice Correspondence from Thomas R. Gatcliffe to Gary J. Bracken, dated December 23, 1998. Subject: "Statistical Review of Technical Basis Document RF/RMRS-98-242 and Further Analysis of Sample Data Restrictions in RFETS Draft Procedure PRO-267-RSP 09.05 (Rev 1)." TRG-040-98.

NUREG 1608/RAMREG-003. "Categorizing and Transporting Low Specific Activity Materials and Surface Contaminated Objects." Research and Special Programs Administration, U.S. Department of Transportation, Washington, DC. 1998.

"Nevada Test Site Waste Acceptance Criteria (NTSWAC), Revision 3." Waste Management Division, Nevada Operations Office, U.S. Department of Energy December 2000. www.nv.doe.gov/programs/envmgmt/rwap/ntswac.htm.

10 CFR 61.55

"Waste Acceptance Guidelines, Revision 2." Envirocare of Utah. March, 2001 www.envirocareutah.com

Multi-Agency Radiation Survey and Site Investigation Manual (MARSSIM), NUREG-1575/EPA 402-R-97-016, December 1997.

- ¹⁰ Ott, W.R., *Environmental Statistics and Data Analysis*, CRC Press LLC, Boca Raton, 1995.

Mayer, A.D. and A.M. Sykes, 1996. "Statistics"; Arnold, London.

Evans, Merran; Nicholas Hastings; and Brian Peacock; 1993. "Statistical Distributions", 2nd ed.; John Wiley & Sons, New York.

A. D. Palachek, Statistical Applications, Internal Report SA-94-007. EG&G Rocky Flats Inc. Golden, Colorado. August 10, 1994.

- ¹⁴ D'Agostino, R.B., A. Belanger, and R.B. D'Agostino Jr. 1990. "A Suggestion for Using Powerful and Informative Tests of Normality.", *The American Statistician*, November, 1990, Vol. 44 No. 4, pp. 316-321.
 - ¹⁵ TBD-00146. "Calculation of Beta Activity for Depleted Uranium, Highly Enriched Uranium and Aged Weapons Grade Plutonium." RFETS Radiological Technical Basis Document.
 - ¹⁶ ANSI N323A-1997. "Radiation Protection Instrumentation Test and Calibration - Portable Survey Instruments." American National Standards Institute. 1997.
- NUREG 1608, Section 3.2.5
- ¹⁸ ISO 7503-1:1988 (E). "Evaluation of Surface Contamination – Part 1: Beta-emitters (maximum energy greater than 0.15 MeV) and alpha emitters." International Organization for Standardization.
 - ¹⁹ "Marks' Standard Handbook for Mechanical Engineers". McGraw-Hill. 1996.
 - ²⁰ T. J. Glover. "Pocket Reference." Sequoia Publishers. 1994.

Appendix A.

Calculating One-Sided Tolerance Limits Based on Weighted Means from Multiple Samples

A. D. Palachek



PalachekUTLpaper.pdf

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CALCULATING ONE-SIDED TOLERANCE LIMITS BASED ON WEIGHTED MEANS FROM MULTIPLE SAMPLES

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Statistical Applications

August 10, 1994

SA-94-007

Statistical Applications

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Internal Report - Not Cleared for Publication

Approved: Dr. Wein

Executive Summary/Introduction:

Methods are provided for calculating a one-sided tolerance limit using a weighted combination of means taken from several independent identical normal distributions. Comparisons are also made between three numerical approximations to the exact tolerance factor constant. The methodology is applicable to determining potential beryllium contamination in a building by weighting the results taken from several rooms.

Discussion:

This methodology was developed to support investigations of potential beryllium contamination in buildings. As discussed by Splett and Weier (1994), statistical confidence statements regarding potential beryllium contamination are to be made on a building-wide basis. Randomly selected measurements of horizontal surfaces will be taken from the rooms within a building. Since the rooms are of different sizes, the results from each room will be weighted using the size of the room to obtain an upper tolerance limit (UTL). Assuming that all assumptions are met, the statistical methodology provides 95% confidence that at least 98% of the horizontal surface in the building has a beryllium contamination level less than this UTL.

The usual tolerance limit methodology is restricted to the situation involving a single sample taken from a normal distribution. The derivation below provides the general formulation for determining an upper (or lower) tolerance limit based upon weighting sample means taken from independent identical normal distribution.

The beryllium measurements are assumed to follow a lognormal distribution, so the logarithms of the measurements follow a normal distribution. In addition, both the means and standard deviations of the measurements must be the same from room to room for the methodology to be applicable. The UTL (U_{log}) is computed for the log-transformed data, and $\exp(U_{log})$ provides the UTL for the original data. Other issues regarding the use of the UTL methodology for detecting building contamination are discussed in Splett and Weier (1994) and Weier and Splett (1994).

Mathematical Derivation:

Let X_{ij} , $j=1,2,\dots,n_i$, denote a sample of n_i observations from the i th distribution, $i=1,2,\dots,r$. Assume that the X_{ij} are normally distributed with mean μ and variance σ^2 for $i=1,2,\dots,r$ and $j=1,2,\dots,n_i$, and also assume that the X_{ij} are independent both within and between samples.

Denote the sample mean and variance from the i th room by

$$\bar{X}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} X_{ij} \quad (1)$$

and $s_i^2 = \frac{\sum_{j=1}^{n_i} (X_{ij} - \bar{X}_i)^2}{n_i - 1}$, respectively.

Let w_1, w_2, \dots, w_r denote the weightings assigned to the r samples, where $0 \leq w_i \leq 1$ for $i=1, 2, \dots, r$ and

$$\sum_{i=1}^r w_i = 1 \quad (3)$$

The weighted estimate of the mean is defined by

$$Y_w = \sum_{i=1}^r w_i \bar{X}_i. \quad (4)$$

As a result of the normality assumptions, Y_w is also a normal random variable with mean μ and variance $\sigma_w^2 = c\sigma^2$,

$$\text{where } c = \sum_{i=1}^r \frac{w_i^2}{n_i}$$

The parameter σ^2 may be estimated by the pooled sample variance, which is given by

$$S_p^2 = \frac{\sum_{i=1}^r (n_i - 1) s_i^2}{N - r} \quad (6)$$

where $N = \sum_{i=1}^r n_i$ is the combined sample size.

As a result of the distributional assumptions,

$$\frac{(N-r) S_p^2}{\sigma^2} \quad (7)$$

has a chi-square distribution with $N-r$ degrees of freedom.

The remainder of the derivation of the tolerance limit follows the same approach described by Owen (1958) to determine the "usual" (unweighted) one-sided tolerance limit constants.

An upper tolerance limit is a value U for which at least 100% of the population is smaller than the limit with confidence $1 - \alpha$. This limit is mathematically denoted as

$$Y_w + k S_p. \quad (8)$$

The factor k is selected to satisfy the probability statement

$$Pr[Pr(X \leq Y_w + kS_p) \geq P] = 1 - \alpha, \quad (9)$$

where X is a random variable observed from any of the r distributions and has a normal distribution with mean μ and variance σ^2 .

This probability statement may be rewritten as

$$Pr \left[\int_{-\infty}^{Y_w + kS_p} \frac{1}{\sigma\sqrt{2\pi}} \exp \left\{ -\frac{(x - \mu)^2}{2\sigma^2} \right\} dx \geq P \right] = 1 - \alpha$$

Let Z_p denote the P th percentile from the standard normal distribution. Then the tolerance probability statement becomes

$$Pr \left[\frac{Y_w + kS_p - \mu}{\sigma} \geq Z_p \right] = 1 - \alpha.$$

This may then be rewritten as

$$Pr \left[\frac{\frac{Y_w - \mu}{\sigma\sqrt{c}} - \frac{Z_p}{\sqrt{c}}}{\frac{S_p}{\sigma}} \geq \frac{k}{\sqrt{c}} \right] = 1 - \alpha,$$

$$\text{or } Pr \left[\frac{\frac{Y_w - \mu}{\sigma\sqrt{c}} - \frac{Z_p}{\sqrt{c}}}{\frac{S_p}{\sigma}} \leq -\frac{k}{\sqrt{c}} \right] = \alpha$$

The random variable

$$\frac{\frac{Y_w - \mu}{\sigma\sqrt{c}} - \frac{Z_p}{\sqrt{c}}}{\frac{S_p}{\sigma}} \quad (14)$$

has a noncentral t distribution with $N - r$ degrees of freedom and

noncentrality parameter

$$\delta = -\frac{Z_p}{\sqrt{c}}$$

Let $t'_{v,\alpha}(\delta)$ denote the α th percentile from a noncentral t random variable with v degrees of freedom and noncentrality parameter δ . Then the probability statement (13) results in

$$k = -\sqrt{c} \cdot t'_{N-r,\alpha}(-Z_p/\sqrt{c}).$$

Using properties of the noncentral t distribution (see Patel, Kapadia, Owen (1976), p.228), the tolerance factor is equivalent to

$$k = \sqrt{c} \cdot t'_{N-r,1-\alpha}(Z_p/\sqrt{c}).$$

The same tolerance factor is used to obtain a lower tolerance limit. The lower tolerance limit is given by

$$Y_w - kS_p.$$

The usual one sample tolerance interval may be viewed as a special case of determining a tolerance limit based on a sample from a single sample. For this situation, the constants become $r=1$, $N=n$, $w_1=1$, and $c=1/n$. The tolerance factor becomes

$$k = \frac{t'_{n-1,1-\alpha}(\sqrt{n} Z_p)}{\sqrt{n}}, \quad (19)$$

which is the usual one sample tolerance factor as given by Owen (1958).

Numerical Approximations:

Calculation of the tolerance factor requires the percentile from the noncentral t distribution. A closed form solution for this percentile does not exist, so numerical approximations and/or algorithms must be used to obtain an approximate value for $t'_{v,\alpha}(\delta)$. The Statistical Analysis System (SAS) software used by Statistical Applications contains an internal function, TINV, that performs a numerical algorithm to approximate the percentile. Application of this approximation when calculating the usual one-sample tolerance factor leads to results that match those published by Hahn (1970).

However, the SAS function TINV can fail for large absolute values of the noncentrality parameter δ . For the tolerance factor, large values of δ are associated with small values of c , indicating large n . Large n also result in larger degrees of freedom.

When TINV cannot calculate the percentile, another approximation must be used. Some of these approximations are discussed in Chapter 14 of Johnson and Kotz (1970). One approximation was developed by Jennett and Welch (1939) and is given by

$$t'_{v,\alpha}(\delta) \approx \frac{\delta b + Z_{1-\alpha} \sqrt{b^2 + (1-b^2)(\delta^2 - Z_{1-\alpha}^2)}}{b^2 - Z_{1-\alpha}^2(1-b^2)}$$

where $b = \sqrt{\frac{2}{v} \frac{\Gamma(\frac{1}{2}(v+1))}{\Gamma(\frac{1}{2}v)}}$.

The Gamma function is defined as

$$\Gamma(t) = \int_0^{\infty} x^{t-1} e^{-x} dx.$$

Another approximation was developed by Johnson and Welch (1940) and is given

by
$$t'_{v,\alpha}(\delta) \approx \frac{\delta + Z_{1-\alpha} \sqrt{1 + \frac{(\delta^2 - Z_{1-\alpha}^2)}{2v}}}{1 - \frac{Z_{1-\alpha}^2}{2v}}$$

The noncentrality parameter for the t percentile for the tolerance factor is given by

$$\delta = \frac{Z_p}{\sqrt{c}}. \quad (24)$$

So a large noncentrality parameter occurs when a small value is obtained for

$$c = \sum_{i=1}^r \frac{w_i^2}{n_i}. \quad (25)$$

The minimum noncentrality parameter is obtained when one sample has sample size 1 and all of the weight is assigned to this room. This leads to $c=1$ and a minimum noncentrality parameter of

$$\delta_{\min} = Z_p. \quad (26)$$

However, the noncentrality parameter could get large if many samples exist with sample sizes of $n_i=1$.

Comparisons of the approximations were made for the particular case of $P = 0.98$ and $\alpha = 0.05$. The evaluations were made for 20, 60, and 100 degrees of freedom. Figures 1 through 3 plot the tolerance factors obtained for varying

values of the noncentrality parameter δ . Notice that the TINV function fails when δ is larger than 16 for these situations. The factor based on the Jennett-Welch approximation is conservative, giving a tolerance factor that is always larger than that for the TINV algorithm. This factor will lead to the largest upper tolerance limit of the three approximations, while the Johnson-Welch approximation gives the smallest.

However, the differences are fairly small in these cases. When $v = 20$ and $\delta = 16$, the k factors from the Jennett-Welch, TINV, and Johnson-Welch approximations are 2.8571, 2.8310, and 2.8159, respectively. The differences between approximations decrease as the degrees of freedom increase.

The SAS procedure PROC CAPABILITY uses the Johnson-Welch approximation to determine tolerance factors for situations in which the TINV function fails. However, the Johnson-Welch approximation leads to smaller factors than those obtained from the TINV function.

The conservative approach is to use the factor based on the Jennett-Welch approximation, which provides the largest upper tolerance limit of the three approximations. This conservative approach may lead to coverage of more than 100% of the population. Another approach is to calculate the interval using the TINV function and use the Jennett-Welch approximation only for cases in which the TINV algorithm fails.

Conclusions:

Under the assumptions given (independent identical normal distributions), a one-sided tolerance limit may be calculated using a weighted combination of the sample means. If a single numerical approximation is desired, Statistical Applications recommends using the Jennett-Welch approximation (equation (20)) which gives conservative intervals.

References:

- Hahn, G.J. (1970). Statistical Intervals for a Normal Population, Part I. Tables, Examples, and Applications, *Journal of Quality Technology*, 2, 115-125.
- Jennett, W.J. and Welch, B.L. (1939). The Control of Proportion Defective as Judged by a Single Quality Characteristic Varying on a Continuous Scale, *Journal of the Royal Statistical Society, Series B*, 6, 80-88.
- Johnson, N.L. and Kotz, S. (1970). *Continuous Univariate Distributions - 2*. Houghton Mifflin, Boston.
- Johnson, N.L. and Welch, B.L. (1940). Applications of the Noncentral t Distribution, *Biometrika*, 31, 362-389.
- Owen, D.B. (1958). *Tables of Factors for One-Sided Tolerance Limits for a Normal Distribution*, Monograph No. SCR-13, Sandia Corporation, Albuquerque, NM.
- Patel, J.K., Kapadia, C.H., and Owen, D.B. (1976). *Handbook of Statistical Distributions*. Marcel Dekker, New York.

Splett, D.M. and Weier, D.R. (1994). *Proposed Statistical Methodology for Sampling and Analysis in Support of Beryllium Studies*, SA/94-003, Internal Report, Statistical Applications, EG&G Rocky Flats Inc., Golden, CO.

Weier, D.R. and Splett, D.M. (1994). *Addendum to Statistical Applications Report SA/94-003 Proposed Statistical Methodology for Sampling and Analysis in Support of Beryllium Studies*, SA/94-005, Internal Report, Statistical Applications, EG&G Rocky Flats Inc., Golden, CO.

Figure 1
 TOLERANCE FACTOR FOR 20 DEGREES OF FREEDOM
 For 95% Confidence of at Least 98% Coverage of Population

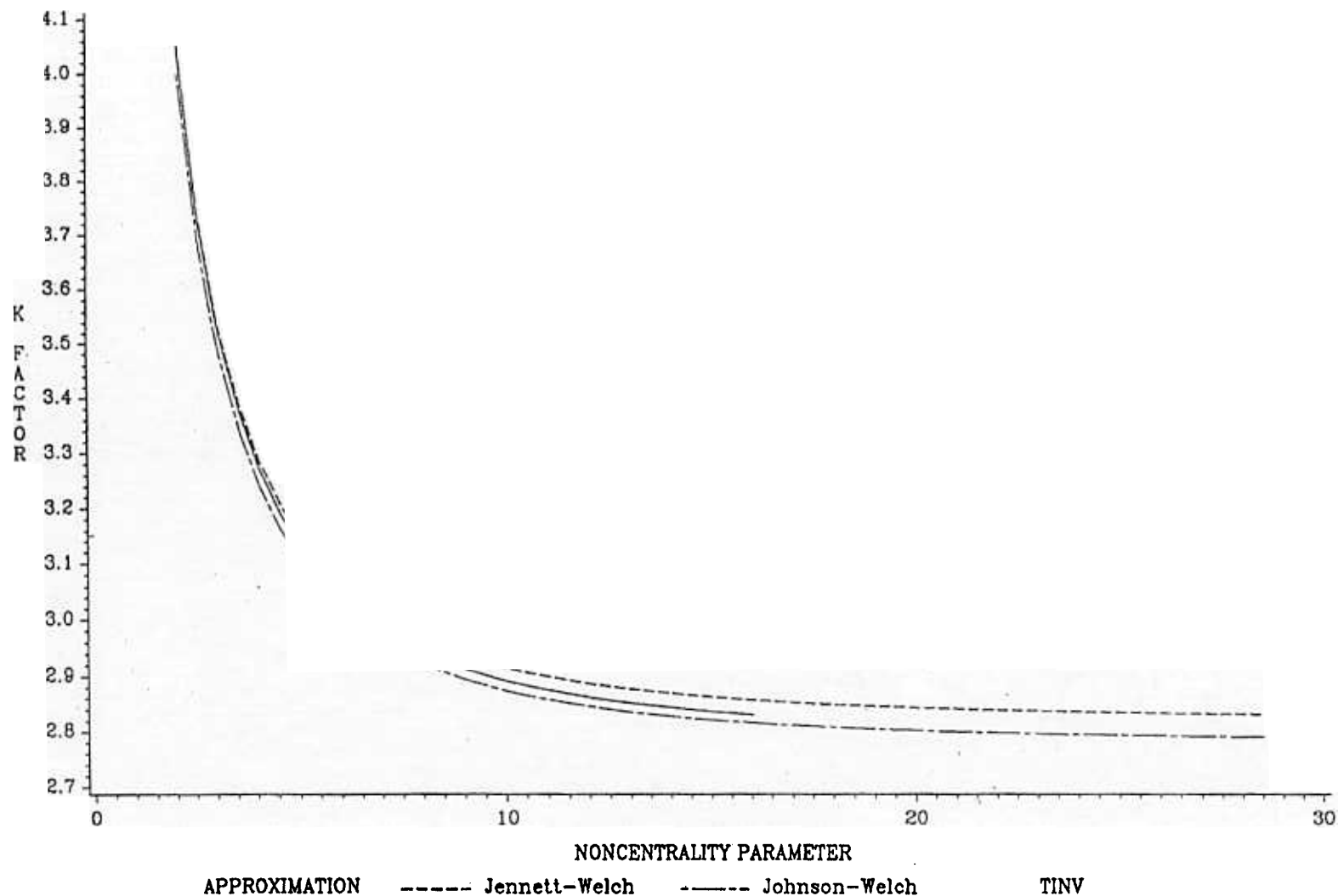


Figure 2
TOLERANCE FACTOR FOR 60 DEGREES OF FREEDOM
For 95% Confidence of at Least 98% Coverage of Population

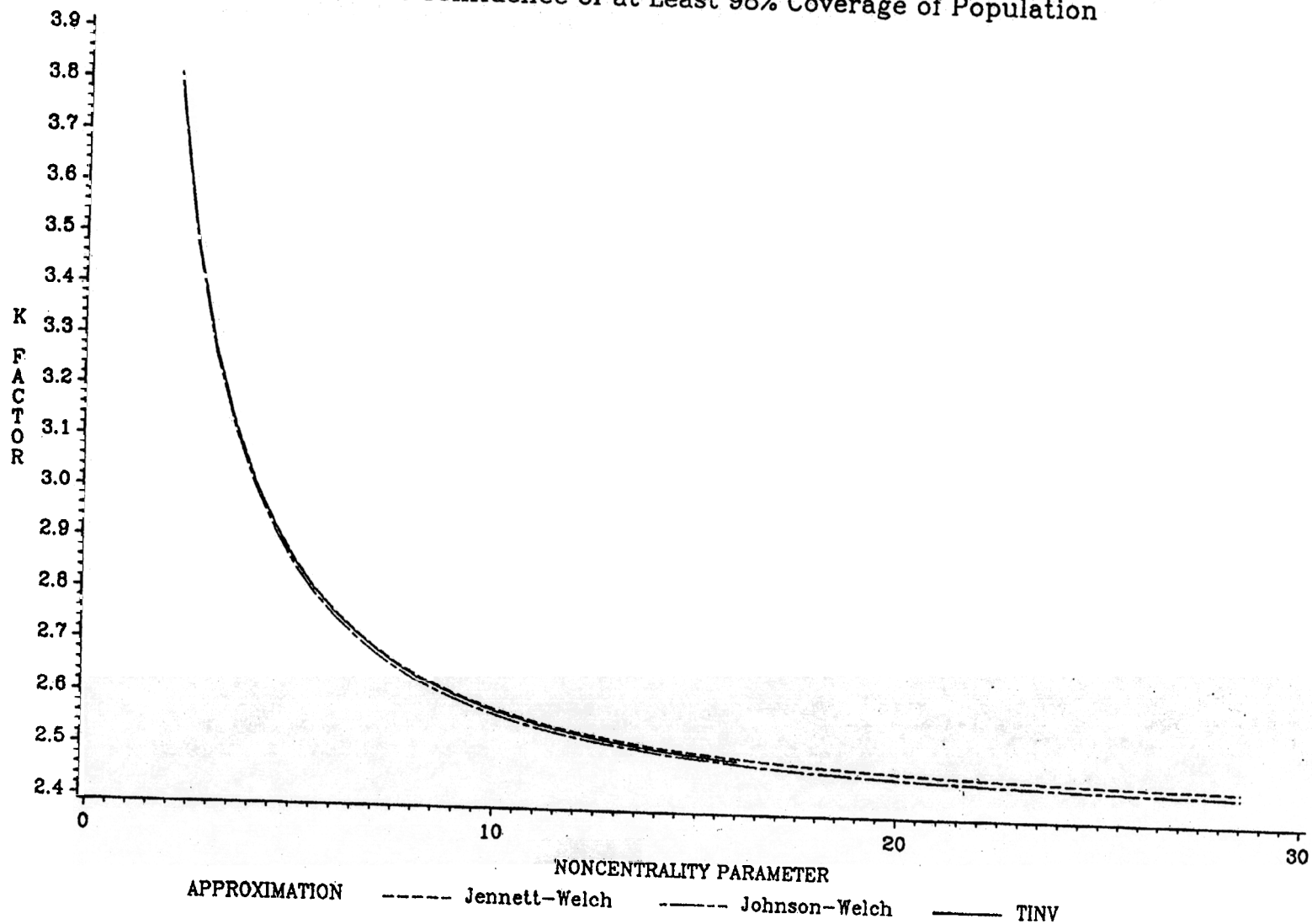
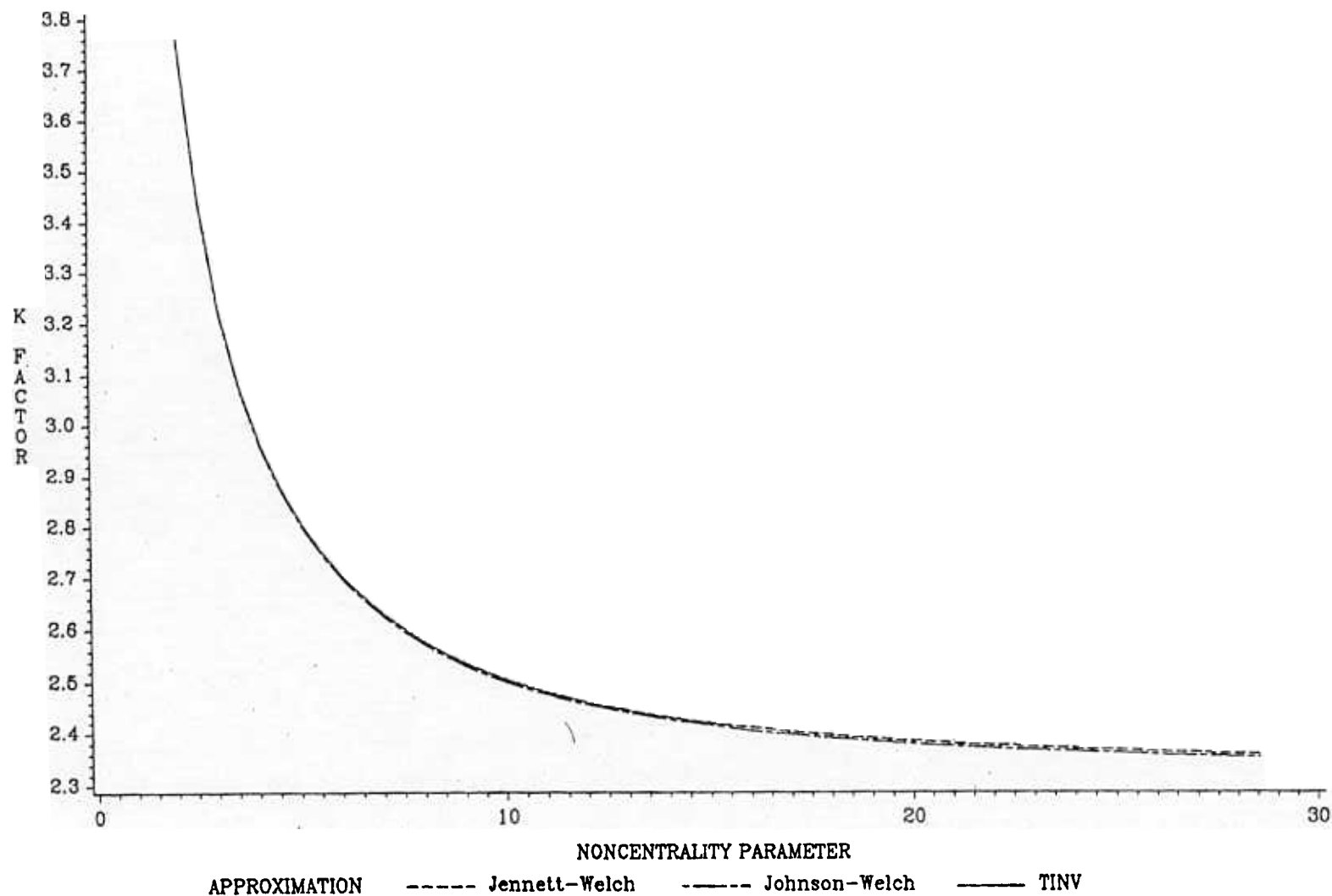


Figure 3
TOLERANCE FACTOR FOR 100 DEGREES OF FREEDOM
 For 95% Confidence of at Least 98% Coverage of Population



Appendix B.

Surface Area to Mass Ratio Data Developed by C. Wallace of Alpha Group, LLC.

Description	Total Surface Area	Weight	SA/MF
Cardboard	17,250 cm ²	408.2 gm	42.26 cm ² /gm
Recycle box			
Plywood	2,080.1 cm ²	907.2 gm	2.29 cm ² /gm
Wooden Boards			
2x4x18 (white pine)	1,153.2 cm ²	589.7 gm	
2x4x18 (yellow pine)	1,153.2 cm ²	771.1 gm	
white & yellow pine ave.SA/MF		680.4 gm ave.	1.695 cm ² /gm
2x6x18 (white pine)	1,644.2 cm ²	861.8 gm	
2x6x18 (yellow pine)	1,644.2 cm ²	1100.0 gm	
white & yellow pine ave.SA/MF		980.9 gm ave.	1.675 cm ² /gm
Printed Circuit Board*			
10 x 6.5 x 3 =	195 cm ²	27.8 gm	
12.8 x 6.5 x 3 =	249 cm ²	113.2 gm	
9 x 7.5 x 3 =	202.5 cm ²	72.2 gm	
19.4 x 21.5 x3 =	1,251.3 cm ²	385.0 gm	
Total Surface Area	1,897.8 cm ²	Total wt. 598.2 gm	3.173 cm ² /gm
*The factor of 3 is use to include the area of the opposite side and the surface area of all the electronic components.			
Computer (Using a "Dell" as average.)			
Tower	7,432 cm ²	14,061.4 gm	
Printed Circuit Board	529.8 cm ²	186.8 gm	
Total Surface Area	1,897.8 cm ²	Total wt. 14,248.2 gm	0.558 cm ² /gm
Monitor (Using a "Dell" as average.)			
16 inch monitor	11,264 cm ²	28,576.3 gm	
Printed Circuit Board	193.5 cm ²	61.0 gm	
Total Surface Area	11,457.5 cm ²	Total wt. 28,637.3 gm	0.400 cm ² /gm
Charles R. Wallace /	/s/		3/15/01
print	sign		Date